

Dynamical Lifshitz-type solutions and aging phenomena

Kunihito Uzawa^{*1} and Kentaroh Yoshida^{†2}

^{}Department of Physics, School of Science and Technology,
Kwansei Gakuin University, Sanda, Hyogo 669-1337, Japan.*

*[†]Department of Physics, Kyoto University
Kyoto 606-8502, Japan.*

Abstract

We consider time-dependent Lifshitz-type solutions in type IIB supergravity. The solutions describe a time evolution from Lifshitz spacetimes to AdS spaces. We argue the holographic relation of them to aging phenomena in condensed matter physics. The solutions have no time-translation invariance and possesses the dynamical scaling symmetry with $z_c = 2$. In addition, the time evolution corresponds to slow-time (non-exponential) relaxation from non-equilibrium states to the equilibrium. We also discuss a mechanism of quantum quench to generate non-equilibrium states.

¹E-mail: uzawa@yukawa.kyoto-u.ac.jp

²E-mail: kyoshida@gauge.scphys.kyoto-u.ac.jp

1 Introduction

The AdS/CFT correspondence [1–3] is supported by an enormous amount of evidence in the present date and there is no doubt for the validity (at least) in the planar limit. An attractive direction in the study of AdS/CFT is to consider its applications to realistic systems such as QCD [4–6] and condensed matter systems [7–11] (For reviews, for example, see [12–14]).

Our purpose here is to consider holographic descriptions of non-equilibrium states dynamically relaxing to the equilibrium state. Since AdS spaces correspond to equilibrium states, the gravitational solutions should be time-dependent and describe a time evolution to AdS spaces. However, it is generally difficult to consider time-dependent AdS/CFT. It is of great importance to understand how to describe non-equilibrium systems holographically in order to obtain the deeper understanding of gravity and AdS/CFT itself.

In this Letter we construct time-dependent Lifshitz-type solutions in type IIB supergravity and argue their holographic descriptions. We show some evidence to support that the solutions describe a class of non-equilibrium phenomena called *the aging* [15–17], which are observed, for example, in glassy materials. The phenomena are characterized by the following three features, 1) no time-translation invariance, 2) the dynamical scaling with the dynamical critical exponent z_c and 3) slow (non-exponential) relaxation. The solutions we construct here satisfy all of them. For earlier holographic approaches to the aging, see [18, 19].

Non-equilibrium states like glassy states are prepared, for example, by quenching the system. There are various ways of quenching. Since we focus upon the zero temperature solutions, a quantum quench, which suddenly change a coupling constant, is concerned with our analysis. We also explain a mechanism of the quantum quench encoded into the gravitational solutions.

Another perspective of the subject is a potential application to realistic cosmological models. The construction of the time-dependent Lifshitz-type solutions is similar to that of dynamical brane solutions [20–22]. These solutions provide the Friedmann-Robertson-Walker universe. According to the similarity, a dynamical universe is obtained from the Lifshitz-type solutions, though there are something undesirable.

2 Dynamical Lifshitz-type solutions

Let us consider D3-brane solutions with waves in type IIB supergravity. We take account of the metric g_{MN} , dilaton ϕ , axion χ and the self-dual five-form field strength $F_{(5)}$. Then the field equations are given by

$$R_{MN} = \frac{1}{2}\partial_M\phi\partial_N\phi + \frac{1}{2}e^{2\phi}F_MF_N + \frac{1}{4\cdot 4!}F_{MA_2\cdots A_5}F_N{}^{A_2\cdots A_5},$$

$$F_{(1)} \equiv d\chi, \quad d[e^{2\phi} * F_{(1)}] = 0, \quad F_{(5)} = *F_{(5)},$$

where $*$ is the Hodge operator in ten dimensions.

We are concerned with a time-dependent generalization of $z_c = 0$ Schrödinger spacetimes in type IIB supergravity and hence we suppose the following ansatz,

$$ds^2 = h^{-1/2}(r) \left[2dudv + G(u, v, r)du^2 + \sum_{i=1}^2 (dy^i)^2 \right] + h^{1/2}(r) \left[dr^2 + r^2 d\Omega_{(5)}^2 \right], \quad (2.1)$$

which is equipped with

$$F_{(5)} = (1 \pm *)d \left[h^{-1}(r) \wedge du \wedge dv \wedge dy^1 \wedge dy^2 \right],$$

$$\phi = \phi_0, \quad F_{(1)} = ku,$$

where ϕ_0, k are constants and $d\Omega_{(5)}^2$ describes the five-dimensional sphere with unit radius. The scalar functions $G(u, v, r)$ and $h(r)$ are not determined yet. The general forms of them are given by, respectively,

$$G(u, v, r) = c_0 + c_1 u + c_2 v + \frac{c_3}{r^4} + \frac{k^2}{4}e^{2\phi_0} \left(\frac{L^4}{r^2} - \frac{r^2}{3} \right),$$

$$h(r) = 1 + \frac{L^4}{r^4}.$$

Here c_a ($a = 0, \dots, 3$) and L are constant parameters. When $c_1 = c_2 = 0$, (2.1) is reduced to the general solutions preserving the $z_c = 0$ Schrödinger symmetry [23]. Non-vanishing c_1 and c_2 break time-translation invariance and phase-rotation symmetry, respectively. Note that c_0 can always be removed by shifting v as $v \rightarrow v - \frac{1}{2}c_0 u$, up to the redefinition of c_1 . Henceforth we will set $c_0 = 0$.

Let us consider the solution (2.1) with $c_1 = c_3 = 0$, for simplicity. Then the near-horizon limit leads to the following metric,

$$ds^2 = \frac{r^2}{L^2} \left[2dudv + F(r, v) \frac{du^2}{r^2} + (dy^1)^2 + (dy^2)^2 \right] + \frac{L^2}{r^2} dr^2 + L^2 d\Omega_{(5)}^2, \quad (2.2)$$

where we have introduced the following quantities,

$$F(r, v) \equiv c_2 r^2 v + \frac{k^2 g_s^2 L^4}{4}, \quad g_s \equiv e^{\phi_0}.$$

Here g_s is the string coupling constant. It is necessary to take the string coupling $g_s \ll 1$ so as to ignore graviton loop effects. This condition is ensured by taking the scaling limit,

$$k \rightarrow \infty, \quad g_s \rightarrow 0, \quad k g_s : \text{fixed},$$

as well as the usual scaling limit,

$$N \rightarrow \infty, \quad g_s \rightarrow 0, \quad N g_s : \text{fixed},$$

where N is the number of D3-branes. Note that $N g_s$ and $k g_s$ are fixed to be two independent constants.

When $c_2 = 0$, the metric (2.2) is the direct product of the $z_c = 0$ Schrödinger spacetime and the five-dimensional sphere S^5 with the unit radius. However, the constant shift symmetry for v is broken when $c_2 \neq 0$. This means that the non-vanishing c_2 breaks the phase rotation in the Schrödinger algebra with $z_c = 0$.

In summary, the metric (2.2) is invariant under i) a constant shift of u , ii) spatial translations of y^i (i=1,2), iii) a rotation on the y^1 - y^2 plane, iv) Galilean symmetries, and v) the scale transformation with $z_c = 0$,

$$u \rightarrow u, \quad v \rightarrow \lambda^2 v, \quad y^i \rightarrow \lambda y^i, \quad r \rightarrow \frac{1}{\lambda} r,$$

where λ is a constant parameter.

It is a turn to derive time-dependent Lifshitz-type solutions by applying the trick in [24] to the solutions (2.2).

The metric (2.2) is first rewritten as

$$\begin{aligned} ds^2 = & -\frac{r^4}{L^2 F(r, v)} dv^2 + \frac{r^2}{L^2} (dy^i)^2 + L^2 \frac{dr^2}{r^2} \\ & + \frac{F(r, v)}{L^2} \left(du + \frac{r^2}{F(r, v)} dv \right)^2 + L^2 d\Omega_{(5)}^2. \end{aligned} \quad (2.3)$$

Since the u -direction is still invariant under a constant shift, one may impose a periodic boundary condition for this direction.

The boundary condition breaks the Galilean symmetries as shown in [24]. Now that the coordinate v can be regarded as the time direction rather than u , the geometry should be

understood as a time-dependent Lifshitz spacetime. The remaining symmetry is close to the aging algebra [25], but special conformal and Galilean symmetries are not contained.

Note that there is a subtlety as the Kaluza-Klein (KK) reduction. This is an intrinsic point to the dynamical case. The radius of compactification is not small everywhere in comparison to the characteristic length-scale L of the solutions. To make matters worse, it even shrinks to zero when $F(r, v) = 0$. Hence the compactification produces another curvature singularity, apart from divergent tidal-forces intrinsic to Lifshitz and Schrödinger spacetime [11, 12, 26, 27]. Indeed, when $F(r, v) = 0$, the KK-gauge field also diverges.

3 The behavior of the solutions

To see the time evolution of the solutions (2.3), we use the following coordinates,

$$r = \frac{L^2}{z}, \quad v = \frac{kg_s}{2}\tau.$$

Then the metric (2.3) is rewritten as

$$ds^2 = \frac{L^2}{z^2} \left[-\frac{d\tau^2}{z^2 f(z, \tau)} + (dy^i)^2 + dz^2 \right] + L^2 d\Omega_{(5)}^2 + \frac{k^2 g_s^2}{4} f(z, \tau) L^2 \left(du + \frac{2}{kg_s z^2 f(z, \tau)} d\tau \right)^2, \quad (3.1)$$

where we have defined the following quantities,

$$f(z, \tau) \equiv 1 + \frac{2\tau}{k^2 g_s^2 z^2 \tau_0}, \quad c_2 \equiv \frac{1}{kg_s \tau_0}.$$

We assume that $c_2 > 0$ hereafter and τ_0 is regarded as a microscopic time-scale later.

The new metric (3.1) describes the Lifshitz spacetime when $f(z, \tau) \simeq 1$. Hence the Lifshitz spacetime dominates for all of the values of $z > 0$ when $\tau \simeq 0$.

We first consider the time evolution for $\tau > 0$. As time progresses, the metric (3.1) tends to deviate from the Lifshitz at $\tau = 0$. It is convenient to divide the spacetime into the two regions at a fixed time τ ,

- i) the near-boundary region $z^2 \ll \frac{1}{k^2 g_s^2} \frac{\tau}{\tau_0},$
- ii) the near-horizon region $z^2 \gg \frac{1}{k^2 g_s^2} \frac{\tau}{\tau_0}.$

In the near-horizon region it still remains to be the Lifshitz spacetime. To evaluate the behavior of the metric in the near-boundary region, we take the $\tau \rightarrow \infty$ limit. Then the function $f(z, \tau)$ is evaluated as

$$f(z, \tau) \simeq \frac{2\tau}{k^2 g_s^2 z^2 \tau_0}.$$

By performing the coordinate transformation,

$$\tau = \frac{\tau'^2}{2k^2 g_s^2 \tau_0},$$

the metric (3.1) is rewritten as

$$ds^2 = \frac{L^2}{z^2} [-d\tau'^2 + (dy^i)^2 + dz^2] + L^2 d\Omega_{(5)}^2 + \left(\frac{L\tau'}{2kg_s z \tau_0} \right)^2 \left(du + \frac{2kg_s \tau_0}{\tau'} d\tau' \right)^2.$$

Thus the usual AdS metric has been reproduced. This implies that the Lifshitz spacetime at $\tau = 0$ tends to decay to the AdS space from the boundary as $\tau \rightarrow \infty$. The time-evolution is shown in FIG. 1. Note that the compactification radius is large enough in the near-boundary region or late time and hence the KK-direction is effectively decompactified in the AdS region.

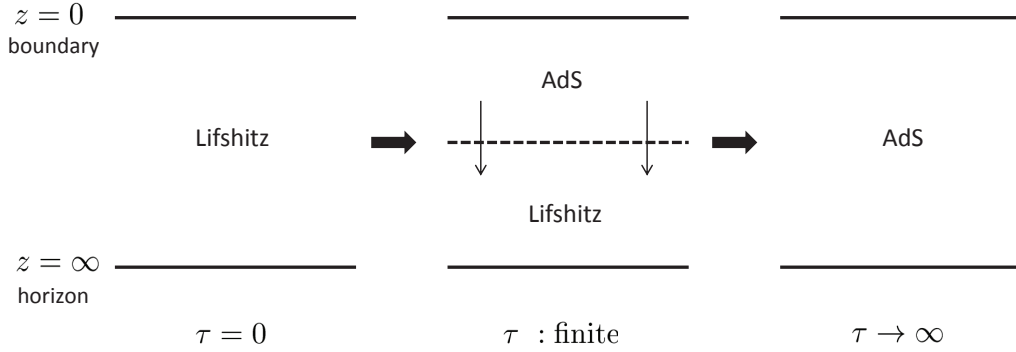


Figure 1: Time evolution of the dynamical Lifshitz solutions.

The next issue is to study the conformal boundary of the solutions (3.1). It is helpful to go back to the coordinate r . Then the metric is expressed as

$$ds^2 = \frac{r^2}{L^2} \left[-\frac{k^2 g_s^2}{2g(r, \tau)} \left(\frac{\tau_0}{\tau} \right) d\tau^2 + (dy^i)^2 \right] + \frac{L^2}{r^2} dr^2 + L^2 d\Omega_{(5)}^2 + \frac{r^2}{2L^2} \left(\frac{\tau}{\tau_0} \right) g(r, \tau) \left(du + \frac{kg_s}{g(r, \tau)} \left(\frac{\tau_0}{\tau} \right) d\tau \right)^2,$$

where we have defined a new function,

$$g(r, \tau) \equiv 1 + \frac{k^2 g_s^2 L^4 \tau_0}{2r^2 \tau}.$$

Then by performing a conformal transformation for the metric as

$$ds^2 \rightarrow \Omega^2(r, \tau) ds^2$$

with the conformal factor,

$$\Omega^2(r, \tau) \equiv \frac{L^2}{r^2} \frac{2g(r, \tau)}{k^2 g_s^2} \frac{\tau}{\tau_0} \quad (\tau > 0),$$

and by taking the limit $r \rightarrow \infty$, the metric becomes

$$ds^2 = -d\tau^2 + \frac{2}{k^2 g_s^2} \left(\frac{\tau}{\tau_0} \right) (dy^i)^2 + \frac{1}{k^2 g_s^2} \left(\frac{\tau}{\tau_0} \right)^2 \left(du + k g_s \left(\frac{\tau_0}{\tau} \right) d\tau \right)^2,$$

and describes a dynamical universe with a power-low expansion. It is a point that the conformal boundary is well-defined differently from the Lifshitz spacetime.

Let us argue what happens in the boundary-theory from the bulk structure. Now the AdS/CFT dictionary tells us that the short-distance physics is dominated by the bulk AdS space and corresponds to equilibrium states. On the other hand, the Lifshitz spacetime influences the long-distance physics and it should be regarded as non-equilibrium states (See FIG. 2). This structure is quite similar to glassy materials, where there exists no long-range order but the ordered structure like a crystalline lattice at short distance. Thus one may expect the relaxation like in the aging phenomena.

The correlation length of equilibrium state depends on time because of the expansion of the universe. The time dependence is evaluated as

$$L(\tau) = \left(\frac{\tau}{\tau_0} \right)^{1/2} L(\tau_0). \quad (3.2)$$

where $L(\tau_0)$ can be evaluated holographically by using the middle point value of the z -direction, $z_{\text{mid}} = 1/kg_s$ as in FIG. 2 and $L(\tau_0)$ depends on kg_s . The time-dependence represents the slow (non-exponential) relaxation with $z_c = 2$ in the aging phenomena, where $L(\tau)$ behaves as $L(\tau) \sim \tau^{1/z_c}$.

Now the physical meaning of τ_0 is also obvious. When $\tau_0 = \infty$, $L(\tau)$ vanishes and hence no relaxation occurs. The whole geometry of the bulk is the Lifshitz spacetime. When $\tau_0 = 0$, $L(\tau) = \infty$ becomes infinite and the system is completely relaxed. Then the

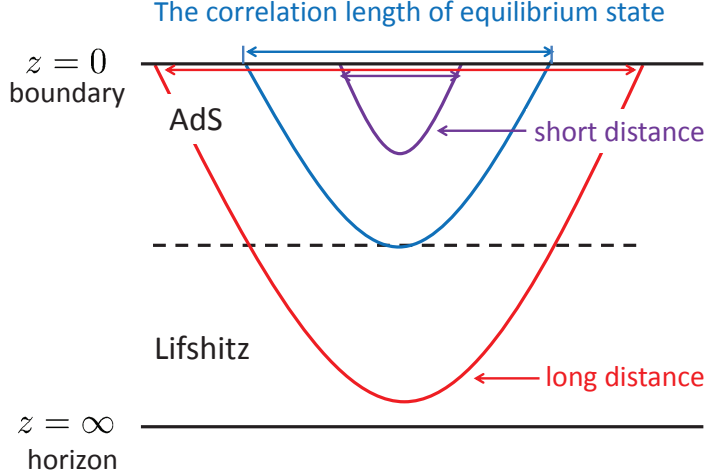


Figure 2: The short-distance physics is dominated by the AdS space, while the long-distance physics is influenced by the Lifshitz spacetime. The correlation length of equilibrium state is estimated with the middle point between the two regions.

whole bulk is the AdS space. Thus τ_0 should be identified with a typical relaxation time at microscopic scale.

Finally let us discuss the behavior of the solutions when $\tau < 0$. Then the solutions (3.1) have a curvature singularity when $f(z, \tau) = 0$. The singularity is located at $z = z_s(\tau)$,

$$z_s(\tau) \equiv \frac{\sqrt{2}}{kg_s} \sqrt{-\frac{\tau}{\tau_0}}, \quad (3.3)$$

and depends on time τ . It appears from the horizon at $\tau = -\infty$ and it goes up to the boundary and finally runs into the boundary at $\tau \rightarrow 0$. The time evolution for $\tau < 0$ is depicted in FIG. 3. In the region where $f(z, \tau) < 0$, the τ -coordinate cannot be regarded as the time direction any more. The metric in this region behaves as a Euclidean AdS (EAdS) space and the u -direction becomes time-like. Now that the u -direction is compactified, a closed time-like exists in this region.

The bump of the singularity may be interpreted as a quantum quench to produce non-equilibrium states such as glassy materials. Indeed, the effective time-dependent coupling $g_{\text{eff}}(\tau)$ defined as

$$kg_{\text{eff}}(\tau) \equiv kg_s \sqrt{\frac{\tau_0}{|\tau|}}, \quad (3.4)$$

diverges rapidly as $\tau \rightarrow 0$ and this may be understood as a holographic description of quantum quench [28].

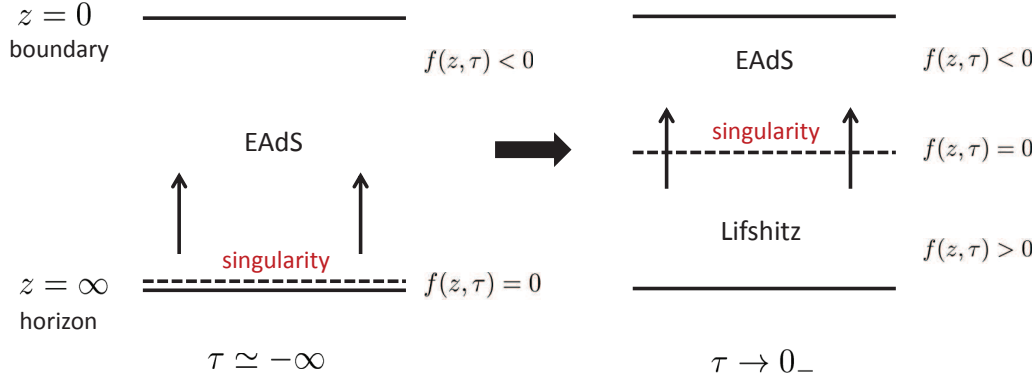


Figure 3: A moving curvature singularity when $\tau < 0$.

4 Conclusion and Discussion

In conclusion, we have shown time-dependent Lifshitz-type solutions in type IIB supergravity and discussed the time evolution. We have also considered the holographic relation to the aging phenomena. It would be possible to obtain more general solutions by turning on various supergravity fields, which are related to more complicated relaxation processes. It is especially interesting to consider the finite-temperature generalization by constructing black hole solutions. It is also interesting to apply our scenario to the logarithmic aging phenomena (For example, see [29]).

As another aspect, our solutions give rise to a dynamical universe with a power-law expansion. This universe is not isotropic because of the KK-circle expanding with time, and hence it would be difficult to construct realistic models with our present solutions. It is interesting to look for other setups applicable to the realistic model building.

We hope that our results develop a new frontier for studying the time-dependent AdS/CFT correspondence and find many applications to non-equilibrium phenomena in condensed matter physics.

Acknowledgments

We would like to thank S. Nakamura for useful discussions. The work of KY was supported by the scientific grants from the Ministry of Education, Culture, Sports, Science and Technology (MEXT) of Japan (No. 22740160). This work was also supported in part

by the Grant-in-Aid for the Global COE Program “The Next Generation of Physics, Spun from Universality and Emergence” from MEXT, Japan.

References

- [1] J. M. Maldacena, “The large N limit of superconformal field theories and supergravity,” *Adv. Theor. Math. Phys.* **2** (1998) 231 [*Int. J. Theor. Phys.* **38** (1999) 1113]. [arXiv:hep-th/9711200].
- [2] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, “Gauge theory correlators from non-critical string theory,” *Phys. Lett. B* **428** (1998) 105 [arXiv:hep-th/9802109].
- [3] E. Witten, “Anti-de Sitter space and holography,” *Adv. Theor. Math. Phys.* **2** (1998) 253 [arXiv:hep-th/9802150].
- [4] G. Policastro, D. T. Son and A. O. Starinets, “The Shear viscosity of strongly coupled N=4 supersymmetric Yang-Mills plasma,” *Phys. Rev. Lett.* **87** (2001) 081601 [hep-th/0104066].
- [5] P. Kovtun, D. T. Son and A. O. Starinets, “Viscosity in strongly interacting quantum field theories from black hole physics,” *Phys. Rev. Lett.* **94** (2005) 111601 [hep-th/0405231].
- [6] T. Sakai and S. Sugimoto, “Low energy hadron physics in holographic QCD,” *Prog. Theor. Phys.* **113** (2005) 843 [hep-th/0412141]; “More on a holographic dual of QCD,” *Prog. Theor. Phys.* **114** (2005) 1083 [hep-th/0507073].
- [7] S. Ryu and T. Takayanagi, “Holographic derivation of entanglement entropy from AdS/CFT,” *Phys. Rev. Lett.* **96** (2006) 181602 [hep-th/0603001]; “Aspects of Holographic Entanglement Entropy,” *JHEP* **0608** (2006) 045 [hep-th/0605073].
- [8] S. A. Hartnoll, C. P. Herzog and G. T. Horowitz, “Building a Holographic Superconductor,” *Phys. Rev. Lett.* **101** (2008) 031601 [arXiv:0803.3295 [hep-th]]; “Holographic Superconductors,” *JHEP* **0812** (2008) 015 [arXiv:0810.1563 [hep-th]].

- [9] D. T. Son, “Toward an AdS/cold atoms correspondence: A Geometric realization of the Schrodinger symmetry,” *Phys. Rev. D* **78** (2008) 046003 [arXiv:0804.3972 [hep-th]].
- [10] K. Balasubramanian and J. McGreevy, “Gravity duals for non-relativistic CFTs,” *Phys. Rev. Lett.* **101** (2008) 061601 [arXiv:0804.4053 [hep-th]].
- [11] S. Kachru, X. Liu and M. Mulligan, “Gravity Duals of Lifshitz-like Fixed Points,” *Phys. Rev. D* **78** (2008) 106005 [arXiv:0808.1725 [hep-th]].
- [12] S. A. Hartnoll, “Lectures on holographic methods for condensed matter physics,” *Class. Quant. Grav.* **26** (2009) 224002 [arXiv:0903.3246 [hep-th]].
- [13] J. McGreevy, “Holographic duality with a view toward many-body physics,” *Adv. High Energy Phys.* **2010** (2010) 723105 [arXiv:0909.0518 [hep-th]].
- [14] S. Sachdev, “Condensed matter and AdS/CFT,” arXiv:1002.2947 [hep-th].
- [15] L. C. E. Struik, “Physical aging in plastics and other glassy materials,” *Polymer Engineering and Science* **17** (1977) 165.
- [16] L. F. Cugliandolo, “Dynamics of glassy systems,” arXiv:cond-mat/0210312.
- [17] M. Henkel and M. Pleimling, “Local scale-invariance in disordered systems,” cond-mat/0703466.
- [18] For an earlier attempt for holographic aging phenomena, see D. Minic and M. Pleimling, “Non-relativistic AdS/CFT and Aging/Gravity Duality,” *Phys. Rev. E* **78** (2008) 061108 [arXiv:0807.3665 [cond-mat.stat-mech]]. However, their setup is invariant under time translation.
- [19] A time-dependent background is proposed as a holographic dual for the aging in J. I. Jottar, R. G. Leigh, D. Minic and L. A. Pando Zayas, “Aging and Holography,” *JHEP* **1011** (2010) 034 [arXiv:1004.3752 [hep-th]]. S. Hyun, J. Jeong and B. S. Kim, “Finite Temperature Aging Holography,” *JHEP* **1203** (2012) 010 [arXiv:1108.5549 [hep-th]]. However, the time dependence can be removed by a coordinate transformation, so the solution is essentially time-independent.

- [20] G. W. Gibbons, H. Lu and C. N. Pope, “Brane worlds in collision,” *Phys. Rev. Lett.* **94** (2005) 131602 [arXiv:hep-th/0501117].
- [21] W. Chen, Z. W. Chong, G. W. Gibbons, H. Lu and C. N. Pope, “Horava-Witten stability: Eppur si muove,” *Nucl. Phys. B* **732** (2006) 118 [arXiv:hep-th/0502077].
- [22] H. Kodama and K. Uzawa, “Moduli instability in warped compactifications of the type IIB supergravity,” *JHEP* **0507** (2005) 061 [arXiv:hep-th/0504193].
- [23] W. Chemissany and J. Hartong, “From D3-Branes to Lifshitz Space-Times,” *Class. Quant. Grav.* **28** (2011) 195011 [arXiv:1105.0612 [hep-th]].
- [24] A. Donos and J. P. Gauntlett, “Lifshitz Solutions of D=10 and D=11 supergravity,” *JHEP* **1012** (2010) 002 [arXiv:1008.2062 [hep-th]].
- [25] M. Henkel, M. Pleimling, C. Godreche and J. -M. Luck, “Aging and conformal invariance,” *Phys. Rev. Lett.* **87** (2001) 265701 [hep-th/0107122].
- [26] K. Copsey and R. Mann, “Pathologies in Asymptotically Lifshitz Spacetimes,” *JHEP* **1103** (2011) 039 [arXiv:1011.3502 [hep-th]].
- [27] G. T. Horowitz and B. Way, “Lifshitz Singularities,” *Phys. Rev. D* **85** (2012) 046008 [arXiv:1111.1243 [hep-th]].
- [28] A holographic quantum quench is argued in another fashion in S. R. Das, T. Nishiooka and T. Takayanagi, “Probe Branes, Time-dependent Couplings and Thermalization in AdS/CFT,” *JHEP* **1007** (2010) 071 [arXiv:1005.3348 [hep-th]]; P. Basu and S. R. Das, “Quantum Quench across a Holographic Critical Point,” *JHEP* **1201** (2012) 103 [arXiv:1109.3909 [hep-th]].
- [29] A. Amir, Y. Oreg and Y. Imry, “Slow Relaxations and Aging in the Electron Glass,” *Phys. Rev. Lett.* **103** (2009) 126403.